3/15/17

"Experience enables you to recognize a mistake when you make it again."-Franklin P. Jones

HW: Test Friday 3/17

AIM: What is an Anti-Derivative?

Warm Up: Find $\frac{dy}{dx}$ of each of the following:

1) $y = x^2 \longrightarrow \frac{dy}{dx} = 2x$

2) $y = 2x^3 \longrightarrow \frac{dy}{dx} = 6x^2$

3) $y = 3x^6 \longrightarrow \frac{dy}{dx} = 18x^5$

4) $y = ax^n \longrightarrow \frac{dy}{dx} = anx^{n-1}$

"a" is a constant

Find $f(x)$ for each of the following:

5) $f'(x) = x^3 \longrightarrow f(x) = \frac{1}{4}x^4$

6) $f'(x) = x^7 \longrightarrow f(x) = \frac{1}{8}x^8$

7) $f'(x) = 2x^8 \longrightarrow f(x) = \frac{2}{9}x^9$

8) $f'(x) = ax^n \longrightarrow f(x) = \frac{a}{n+1}x^{n+1}$

"a" is a constant

$f(x)$ is called the anti-derivative of $f'(x)$

$F(x)$ → Anti-derivative of $f(x)$

$f(x)$ → derivative of $F(x)$

$f'(x)$ → derivative of $f(x)$
Second derivative of $F(x)$

$f''(x)$ → derivative of $f'(x)$
Second derivative of $f(x)$
third derivative of $F(x)$

Each function has a family of functions that represent the Anti-derivative.

Ex) $f(x) = 3x^2 + 2$ $\quad g(x) = 3x^2 + 9$

$f'(x) = 6x$ $\quad g'(x) = 6x$

$3x^2 + 2$ and $3x^2 + 9$ are members of the family of functions whose Antiderivative is $6x$

If $f(x) = 4x^5 - 6x + 4x^0$, $F(0) = -1$ (initial condition)

then find $F(x)$ (Anti-derivative of $f(x)$)

$$F(x) = \frac{4x^6}{6} - \frac{6x^2}{2} + 4x + c$$ ← constant of integration

$$F(x) = \frac{2}{3}x^6 - 3x^2 + 4x + c$$ ← General Solution

use the initial condition to solve for c

$$-1 = \frac{2}{3}(0)^6 - 3(0)^2 + 4(0) + c$$

$$-1 = 0 - 0 + 0 + c$$

$$-1 = c$$

$$F(x) = \frac{2}{3}x^6 - 3x^2 + 4x - 1$$ ← Specific solution that satisfies the initial condition.

Given: $f'(x) = 8x^3 + 12x + 3$, $f(1) = 6$

Find: $f(x)$

$$f(x) = \frac{8x^4}{4} + \frac{12x^2}{2} + 3x + c$$

$$f(x) = 2x^4 + 6x^2 + 3x + c$$

$$6 = 2(1)^4 + 6(1)^2 + 3(1) + c$$

$$6 = 2 + 6 + 3 + c$$

$$6 = 11 + c$$

$$-5 = c$$

$$f(x) = 2x^4 + 6x^2 + 3x - 5$$

Given: $f''(x) = \sqrt{x} + 3$ $f'(0) = -3$

Find: $f'(x)$

$\sqrt{x} + 3 \rightarrow x^{1/2} + 3$

$$f'(x) = \frac{x^{3/2}}{3/2} + 3x + c$$

$$f'(x) = \frac{2}{3}x^{3/2} + 3x + c$$

$$-3 = \frac{2}{3}(0)^{3/2} + 3(0) + c$$

$$-3 = c$$

$$f'(x) = \frac{2}{3}x^{3/2} + 3x - 3$$